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- Likewise, the limit $\lim_{(x,y)\to(a,b)} f(x,y)$ of a function f(x,y) as (x,y) approaches the point (a, b) measures the behavior of f(x, y) when the distance $\sqrt{(x-a)^2 + (y-b)^2}$ from (x,y) to (a,b) is very small.

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One other way to see it is to convert to polar coordinates. We have that $x = r \cos \theta$ so that by the constant rule for limits, we conclude that

$$\lim_{(x,y)\to(0,0)} x = \lim_{r\to 0} (r\cos\theta) = \cos\theta \cdot (\lim_{r\to 0} r) = 0.$$

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Consequently, for different choices of $m \neq 0$, we obtain a different limit.

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- Composition of continuous functions preserves continuity.
- Polynomials, rational functions, power functions, trigonometric functions, inverse trigonometric functions, exponential functions, logarithmic functions, and any composite of these are continuous.

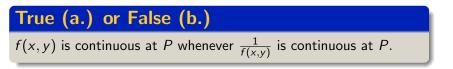
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(a.) True. Compositions of continuous functions are continuous. Given that $\frac{1}{f(x,y)}$ is continuous at P, we have that f(x,y) is continuous at P since $\frac{1}{x}$ is continuous for $x \neq 0$ and $f(x,y) = \frac{1}{x} \circ \frac{1}{f(x,y)}$.

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- Strategy: Given that all else has failed, convert to polar or cylindrical coordinates. If the limit depends on the angle θ, then it does not exist; otherwise, the limit exists and equals the computed number.

True (a.) or False (b.)

Given that f(x, 0) = 3 for $x \neq 0$ and f(0, y) = 5 for $y \neq 0$, we have that $\lim_{(x,y)\to(0,0)} f(x, y) = (3, 5)$.

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(a.) False. Consider the piecewise function

$$f(x,y) = \begin{cases} 0, & \text{if } x = 0, y = 0; \\ 3, & \text{if } x \neq 0, y = 0; \\ 5 & \text{if } x = 0, y \neq 0; \text{ and} \\ \frac{1}{x+y} & \text{if } x \neq 0, y \neq 0. \end{cases}$$

On the path y = x, $\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{x\to 0} \frac{1}{2x}$ does not exist.

Computing Limits in Two Variables

Compute the limit $\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+y^2}$.

(a.) 1 (c.)
$$\frac{1}{3}$$

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We first plug and chug to find the indeterminate form $\frac{0}{0}$. We try the xand y-axes to find that the limits both exist and equal 0. Using the path y = mx, however, yields $\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+y^2} = \lim_{x\to 0} \frac{mx^2}{(3+m^2)x^2} = \frac{m}{3+m^2}$.

1