# MATH 126: Inverse Trigonometric Functions

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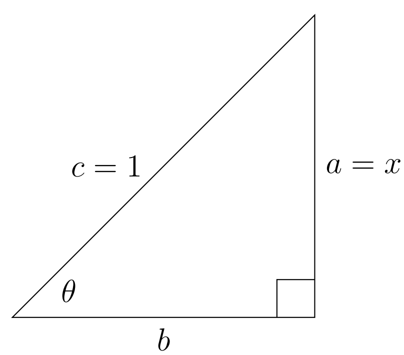
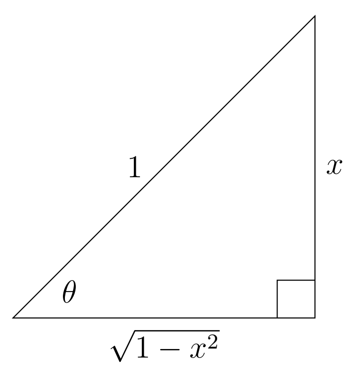
We will compute the derivatives and antiderivatives of the inverse trigonometric functions. We will first set up a right triangle with $\sin\theta = x$. By definition, we have that $\arcsin x = \theta$ so that

$$\frac{d}{dx}\arcsin x = \frac{d\theta}{dx}.$$

Observe that $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$, so we may declare that opposite $= x$ and hypotenuse $= 1$ in order to obtain $\sin\theta = x$. Our triangle therefore looks like the following.




By the Pythagorean Theorem, we have that $a^2 + b^2 = c^2$, hence we must have that $b = \sqrt{1 - x^2}$.




Using the Chain Rule, we can compute $\frac{d\theta}{dx}$. Explicitly, we have that

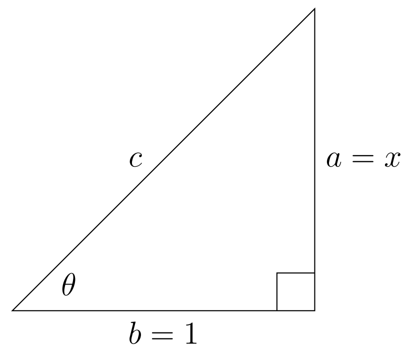
$$\cos\theta \cdot \frac{d\theta}{dx} = \frac{d}{dx}\sin\theta = \frac{d}{dx}x = 1 \text{ so that}$$

$$\frac{d}{dx}\arcsin x = \frac{d\theta}{dx} = \frac{1}{\cos\theta} = \frac{1}{\sqrt{1 - x^2}}.$$

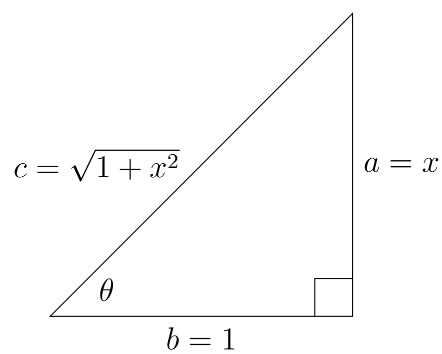
**Example 1.** Use a right triangle involving 1, $x$, and $\sqrt{1-x^2}$ to compute $\frac{d}{dx}\arccos x$.

*Solution.* Blah, blah, blah. ◇

Using a similar idea as the one we employed to compute the derivative of $\arcsin x$ and $\arccos x$, we will set up a triangle with $\tan\theta = x$. Observe that $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$, so we may declare that opposite $= x$ and adjacent $= 1$. Our triangle therefore looks like the following.




Once again, by the Pythagorean Theorem, we find that $c = \sqrt{1+x^2}$.




Using the Chain Rule, we can compute $\frac{d\theta}{dx}$. Explicitly, we have that

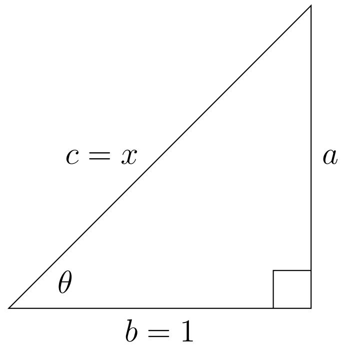
$$\sec^2\theta \cdot \frac{d\theta}{dx} = \frac{d}{dx}\tan\theta = \frac{d}{dx}x = 1 \text{ so that}$$

$$\frac{d}{dx}\arctan x = \frac{d\theta}{dx} = \cos^2\theta = \frac{1}{1+x^2}.$$

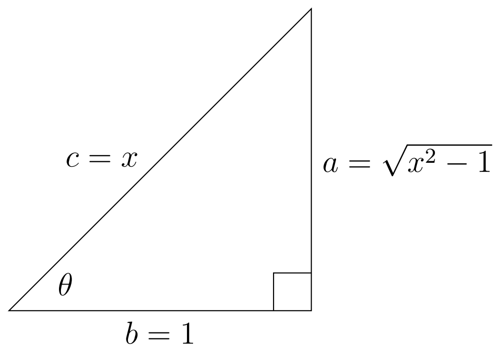
**Example 2.** Use a right triangle involving 1, $x$, and $1+x^2$ to compute $\frac{d}{dx}\operatorname{arccot} x$.

*Solution.* Blah, blah, blah. ◇

Last but not least, we will set up a triangle with $\sec\theta = x$. Observe that $\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$, so we may declare that hypotenuse $= x$ and adjacent $= 1$. Our triangle therefore looks like the following.




Once again, by the Pythagorean Theorem, we find that $a = \sqrt{x^2 - 1}$.




Using the Chain Rule, we can compute $\frac{d\theta}{dx}$. Explicitly, we have that

$$\sec\theta \tan\theta \cdot \frac{d\theta}{dx} = \frac{d}{dx}\sec\theta = \frac{d}{dx}x = 1 \text{ so that}$$

$$\frac{d}{dx}\operatorname{arcsec} x = \frac{d\theta}{dx} = \cos\theta \cot\theta = \frac{1}{x\sqrt{x^2 - 1}}.$$

**Example 3.** Use a right triangle involving 1, $x$, and $1 + x^2$ to compute $\frac{d}{dx}\operatorname{arccsc} x$.

*Solution.* Blah, blah, blah. ◇