MATH 126: Exponential and Logarithmic Functions

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Fall 2020

We will compute the derivatives and antiderivatives of the exponential and logarithmic functions. We will assume that the derivatives

$$\frac{d}{dx}e^x = e^x \text{ and } \frac{d}{dx}\ln|x| = \frac{1}{x}$$

are known. Given a real number $a > 0$, the exponential function $f(x) = a^x$ is differentiable for all real numbers x. Further, observe that $y = a^x$ is strictly positive for all x, hence the function $\ln y = x \ln a$ is well-defined. Using the Chain Rule, we find that

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{d}{dx}\ln y = \frac{d}{dx}(x \ln a) = \ln a \cdot \frac{d}{dx}x = \ln a \text{ so that}$$

$$\frac{d}{dx}a^x = \frac{d}{dx}y = \frac{dy}{dx} = y \ln a = a^x \ln a.$$

Example 1. Compute the derivative of $y = \log_a x$ by using the fact that $a^y = x$.

Solution. Blah, blah, blah. $\diamond$